# Exercise Boundary Violations in American-Style Options:

# The Rule, not the Exception

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## Abstract

An exercise boundary violation (EBV) occurs when the best bid price for an American option is below the option's intrinsic value. Using intraday data, we show that EBVs are actually very common. In March 2010, 48.6% of all bid quotes for in-the-money calls were below intrinsic value, rising to nearly 100% for deep in-the-money contracts near to expiration. Under these conditions, the rational response of an investor liquidating an option is to exercise the option rather than sell it, and we find that EBVs are strong predictors of the likelihood of early exercise. Our results reverse standard theory on American option valuation and optimal exercise strategy. Early exercise is very often rational in the real world and an American option on a non-dividend paying stock should therefore command a premium over the equivalent European contract because of its greater liquidity. Our results also suggest that it makes sense to pay attention to EBVs in estimating implied volatility, marking option positions to market, and measuring market liquidity, perhaps by using an option's intrinsic value in place of an EBV bid quote.



Keywords: American options; option exercise; early exercise boundary; investor rationality

JEL Classifications: G13, G12, G14

Is it rational to exercise an American call option on a non-dividend paying stock before maturity? The conventional wisdom, following Merton (1973), is that early exercise is irrational because it yields only the call's intrinsic value (S – K) and forfeits the option's time value. In this paper, we show unambiguously that trading frictions cause this theoretical result to fail completely in practice. Specifically, for more than 40% of in-the-money calls and puts, the best bid price available in the market is consistently lower than the option's intrinsic value, even with months remaining to expiration. We call this situation an Exercise Boundary Violation (EBV). In an EBV, the holder of a call option cannot liquidate the position by selling in the market at its theoretical value, or even at intrinsic value. An American call, however, can always be exercised to recover at least intrinsic value.

The implications of these results go well beyond the narrow issue of whether investors are rational about option exercise. The theoretical result that American exercise contributes no economic value for a call option except to allow exercise just before a stock goes ex-dividend does not hold in the real world. When EBVs are the rule not the exception, the buyer of a European call must expect that if he wishes or needs to exit the position before expiration and the option is in-the-money, he will lose the remaining time value and will also have to give up an additional amount equal to the EBV in the market at the time. An American option holder would have to give up the time value too, but not the EBV. Thus, regardless of dividend payout an American option is worth more than a European call with the same terms. The value of American exercise in the real world is positive and it arises from trading frictions that are absent from the theoretical model. It can be thought of as a kind of liquidity premium.

Here is a typical example of an EBV. At 1:57.29 p.m. on March 11, 2016, General Electric stock was quoted at $30.36 bid and $30.37 ask. Across all the option trading venues, the best

bid and offer for the GE call option with a $28.00 strike price and seven days to expiration were $2.28 bid and $2.54 ask. At $0.08 below the call's intrinsic value of $2.36, the bid price was an EBV. This particular example was not some odd case. The underlying stock was very actively traded and the open interest for this option was 21,248 contracts.

A similar analysis of EBVs applies to put options. Like calls, American puts have positive optionality value, but the time value of money effect is negative: waiting until maturity gives up interest that could be earned by exercising immediately and investing the proceeds at the riskless rate. Thus, even in a frictionless market, a deep in-the-money American put may be exercised rationally before expiration, although with interest rates close to zero this situation has been fairly rare in recent years. Nevertheless, if a position must be liquidated when the best bid in the market is below intrinsic value, an American put should be exercised, even when that would not be rational in a frictionless world.

Evidence of boundary violations has been noted in studies dating back at least to Klemkosky and Resnick (1979). Despite this long history, much of the research starts from the theoretical premise that option exercise before maturity is irrational. Our paper is the first to systematically examine the size, extent, and persistence of exercise boundary violations that can make early exercise rational, and which therefore make an American option more valuable in practice than a European option with the same terms.

Our data sample is drawn from the full set of intraday quotes and trades for all U.S. equity options during March 2010 and consists of minute-by-minute observations on 2945 stocks and approximately 125,000 individual options. We find EBVs are common, economically large, and very hard for a trader to avoid. Overall, during this period 48.6% of all in-the-money call option

bid quotes were below intrinsic value, by an average of $0.31, with comparable figures for puts. For calls or puts that were in-the-money by 10% or more with less than a month to maturity, over 97% of the quotes were exercise boundary violations. Most were large enough that even a retail investor could save significant amounts after transactions costs by exercising in-the-money options rather than selling them in the market.

When the best market bid is an EBV, a trader has three possibilities to liquidate an in-the-money call option before expiration. First, she can simply sell at the available bid. In-the-money options often have a substantial embedded profit, which may make the holder more willing to accept a modest opportunity loss by selling at an EBV price. Exercise is the second possibility. To lock in the intraday intrinsic value at the moment the decision is made, she can sell short the stock and then use the shares that are received in the exercise to cover the short position. This somewhat more complicated trade involves extra transactions, but it is easy to implement with today's relatively low trading costs. Moreover, brokerage firms often have rules that allow shares acquired through exercise to cover a simultaneous sale in the stock market. In either case, exercise typically dominates selling at the bid. In an article for retail options traders, Yates (2010) explains the problem of EBVs and suggests exactly this liquidation strategy to get around them.[1]

The third possibility is the one envisioned in theory for a frictionless world with no secondary market for options, which is to continue holding the option but delta hedge the position to eliminate the stock price risk. This strategy entails transactions costs to buy and sell the stock

[1] Yates assumes call exercise will place the stock into the trader's account in time to complete the trade, which makes selling the stock an ordinary sale rather than a short sale.

and margin requirements on the short stock position that must be maintained through option expiration. Moreover, frequent rebalancing of a delta hedge in the real world is costly and does not fully eliminate risk. Jensen and Pedersen (2015) explore this strategy in detail and show that European call prices can be driven below intrinsic value by transactions costs, primarily those associated with short sales and funding long positions. That is, while in theory the delta hedging strategy locks in both the option's intrinsic value and its time value, with transactions costs it frequently fails to recover even intrinsic value in practice.

The rest of the paper is structured as follows. Section 2 gives a brief review of the literature. Our data is described in Section 3. Section 4 presents the data on exercise boundary violations for calls and puts. It shows unambiguously that EBVs are pervasive, economically large and highly persistent throughout the trading day.

Section 5 looks at option trades. We find that relatively few trades occur at EBV prices and that the average EBV associated with these trades is comparatively small. Even so, the total amount of money "left on the table" through sub-optimal trading is material. An upper bound estimate is about $39 million for the month. Interestingly, EBV trades are somewhat larger than the average in size.

Section 6 looks at the option holder's alternative liquidation strategy, exercise. We find that non-dividend related early exercise is relatively common among both customers and market makers, although the frequency of early exercise is greater for the market makers. We then fit a logit model to look more directly at the factors that determine the probability of early exercise. Exercise by both customers and market makers is highly dependent on the size and duration of

EBVs, along with option moneyness. Exercise of puts is also strongly influenced by whether the stock price is above or below the put option theoretical early exercise boundary.

In Section 7 we discuss several ways in which pervasive EBVs may affect options theory and practice, including how American options should be priced in theory and how option positions should be marked-to-market in practice, as well as correct calculation of implied volatilities and use of the bid-ask spread as a measure of market liquidity. Section 8 concludes.

An important issue that we do not attempt to address in the current paper is why the best bid prices in the market are so frequently below fair value, to the point that they entail an arbitrage profit for the buyer. Competition among dealers and other liquidity providers ought to force the bids up, but it does not. Yet, discussions with options market makers did not uncover any hidden transactions costs or major impediments to option exercise. We also do not try to derive an alternative American option pricing model to take proper account of the liquidity value of being able to exit an option position early without giving up an EBV. These important topics are left for future research.

## 2. Boundary Violations and Option Exercise in the Literature

Early option market studies such as Bhattacharya (1983) noted the occurrence of exercise boundary violations. Bates (1995) summarizes this literature. More detailed analyses by Diz and Finucane (1993), Finucane (1997) and Engstrom (2002) uncovered boundary violations in U.S. stock index options, U.S. equity options, and Swedish equity options.

Stock and options market structure and liquidity have changed significantly since these studies were done and they were also limited to analyzing closing prices, so they do not reflect market conditions within the trading day. Indeed, Battalio and Schultz (2006) show that reported

closing option prices often seem anomalous and may even show arbitrage violations, but most of the violations appear to be due to non-synchronous reporting of stock and option quotes. A major advantage of our intraday tick dataset is that we are able to look at market conditions throughout the day.

If an option holder can always realize its theoretical value by selling (or delta hedging) in the market, an American call is only better than a European call because it allows exercise just before a stock goes ex-dividend. But when Valkanov, Yadav, and Zhang (2011) and Dueker and Miller (2003) tested this theoretical principle in option market prices, they found early exercise premiums of 3-6% for call options and 4-10% for put options, much too large to be explained by dividend-related early exercise alone.[2] Option traders apparently believe American exercise has some additional value unrelated to dividends.

Most of the academic empirical studies of option exercises have focused on the rationality of the decisions, not on boundary violations. In recent work, Pool, Stoll, and Whaley (2008) and Hao, Kalay, and Mayhew (2010) examine the relation between dividends and exercise behavior. In theory, an in-the-money American call should be exercised just before the ex-dividend date if the dividend payment is sufficiently large, but these studies find that about 50% of the calls for which this would be optimal are not exercised. Barraclough and Whaley (2012) provide a comprehensive analysis of option exercise. They find, as we do, that call options are frequently exercised early for reasons unrelated to dividends, yet some deep in-the-money put options fail to be exercised early. Poteshman and Serbin (2003) document a tendency for option investors to

[2] See also Jorion and Stoughton (1989), McMurray and Yadav (2000), Sung, (1995), Unni and Yadav (1998), and Zivney (1991).

exercise call options when they would have received more by selling the call. This is the flip side of our observation that some investors sell their options at EBV prices when they would have done better by exercising.

The underlying theme in this line of research is that traders often behave irrationally with respect to early exercise of American options. Our position is the opposite: We argue that once EBVs and other market frictions are taken into account, much of the behavior that appears irrational in theory can be seen to be rational in practice. Moreover, this has important implications for the way we should think about the value, or lack thereof, of American exercise.

Recent research is more supportive of the idea that market frictions can make it rational to deviate from conventional early exercise rules. In a theoretical model, Duffie, Liu, and Poteshman (2005) formalize the intuition that an American option should be exercised if the price falls below intrinsic value. But they note that an investor's welfare may be higher by retaining the option if markets are incomplete and trading costs are sufficiently large.

Costly hedging plays a key role in the models of Figlewski (1989) and especially Jensen and Pedersen (2015). Figlewski simulates an options market with rebalancing costs and shows that a bid-ask spread sufficient to compensate the market maker for the risk in hedging his position can lead to bid prices below intrinsic value. Jensen and Pedersen (2015) develop and test a formal continuous-time model for call options in which the costs associated with short sales and funding long positions can drive prices below intrinsic value. Their empirical results indicate that these costs are large enough in practice to make early exercise often the optimal liquidation strategy, and that their model can explain some of the observed "non-rational" early exercise.

Jensen and Pedersen and most other researchers analyze only closing prices and quotes. The results are informative, of course, but as Dennis and Mayhew (2009) found, tests of option pricing models are sensitive to microstructure issues, and Battalio and Schultz (2006) and others have shown that wider spreads and odd quotes are relatively common in end-of-day prices. Moreover, closing prices alone do not reveal what trading possibilities may have been available earlier in the day. Use of tick data allows us to measure precisely the size and persistence of exercise boundary violations holders of both calls and puts face throughout the trading day.

## 3. Data

Our sample covers options on 2945 stocks across all expiration months for 21 trading days during March 2010. Data from March 22 are corrupt and excluded from the analysis. We limit the sample to options that are in-the-money at the start of the trading day and all ETFs are excluded. We also exclude all observations on the day of expiration. Each stock will typically have both call and put options with multiple exercise prices, which yields an average of 124,652 individual options for each trading day.

The data come from four sources. Option trades and quotes are from Livevol. Trade data include the date, time, and size of each transaction. For option quotes, Livevol provides the national best bid and offer prices across the eight option exchanges, reported at one-minute intervals. There are 390 trading minutes per day for each option, which yields a data matrix of roughly 1.02 billion elements: 124,652 (options) x 21 (days) x 390 (minutes). The specific contracts in the sample change over time as option moneyness varies with stock price changes, and expiration dates pass, and not every option series is quoted every day. In addition, we

eliminate very low liquidity contracts with fewer than 180 daily quotes and all observations with a bid price of zero. This reduces our data matrix to approximately 671 million elements.

Intraday tick data for equity trades and quotes to match the options sample come from the NYSE TAQ database. We use CRSP to identify and exclude ex-dividend days. In order to focus exclusively on "irrational" exercise, we eliminate all observations for dividend paying stocks on the day proceeding the ex-dividend day. Finally, data on daily open interest and exercises from both customer and market maker accounts come from the Option Clearing Corporation.[3]

To reduce the dimensionality of the option quotes, we classify the strike price / stock price ratio into three moneyness categories:

"Deep ITM" ($X/S < 0.7$ for calls and $1.3 < X/S$ for puts);

"Mid ITM" ($0.7 < X/S < 0.9$ for calls and $1.1 < X/S < 1.3$for puts); and

"Near ITM" ($0.9 < X/S < 1.0$ for calls and $1.0 < X/S < 1.1$ for puts).

Similarly, we set up three maturity categories: "near term" (1 month or less); "medium term" (2-4 months); and "long term" (over 4 months). Within this 3 x 3 x 2 partition, the number of observations per cell ranges from 12.1 million to 69.9 million.

Note that for an option falling into one of these categories, the likelihood that the option will remain in-the-money at expiration depends on option maturity and the volatility of the underlying stock. For example, a one-month call on a stock with $X / S = 0.9$ and annual volatility of 40% is in-the-money by less than one standard deviation. If $X / S = 0.8$, the

[3] The OCC data were generously provided to us by Robert Whaley of Vanderbilt University.

midpoint of the Mid ITM bracket, a three-month call on that stock is still only 1.1 sigmas in-the-money. Prices for these options should embed substantial optionality, but as we show below, they still have a strong chance of being subject to EBVs.

Historically, options priced below $3.00 traded on 'tick' size of $0.05 and options priced above $3.00 had a tick size of $0.10. In 2007, option exchanges initiated the penny pilot program in which they reduced the tick size for more liquid options.[4] In our sample, 178 option classes were included in the penny pilot program. For these options the tick size was reduced to $0.01 for prices below $3.00, and $0.05 for options above $3.00.

## 4. Frequency, magnitude and persistence of boundary violations

In this section we explore how common EBVs are as a function of moneyness and maturity, how large they are economically, and how persistent they are in terms of how long an option holder facing an EBV bid at the beginning of the day would have to wait to see a bid at or above intrinsic value.

### 4.1 Frequency and Size of Boundary Violations

Table 1 presents summary statistics for exercise boundary violations in our sample. Panel (a) shows results for call options and Panel (b) presents puts. The largest numbers of quotes are for the mid moneyness and middle maturity bins, largely due to the width of the categories, but both panels in Table 1 show high quote frequency for all categories, with the smallest cell containing more than 20 million calls (12 million puts).

[4] Saraoglu, Louton, and Holowczak (2014) examine the impact of the penny pilot program on options trading costs.

For near term Deep ITM calls, 98.73% of all bids were below intrinsic value and 99% of all options in this category had EBV bids at least some of the time. For near term Mid ITM contracts, still 92.77% of all bid quotes were EBVs and more than 98% of all contracts had them. Even for near maturity calls that were only a little in-the-money, the best bid in the market was below intrinsic value more than a third of the time.

The frequency of boundary violations falls with longer maturity and lower moneyness as one would expect. Both factors increase an American option's time value, so even if a bid is well below a long-dated option's theoretical fair value it is less likely to be below intrinsic value and cause an EBV. Even so, for Deep ITM long maturity calls, about 60% of market bid quotes are EBVs. Interestingly, EBVs are somewhat less common for puts, particularly as maturities lengthen, even though in theory puts may be rationally exercised early because their theoretical time value can fall to zero.

Table 1 also provides the mean boundary violation in each maturity and moneyness bucket. We report the unconditional EBV, which is the average of EBVs across all observations, including a value of zero for all non-EBV observations. We also report the conditional EBV, which is the average EBV across all quotes that are EBVs. For comparison, Table 1 also contains the average option bid-ask spread.

A useful comparison is to contrast the EBVs with the quoted bid-ask spread. While there is variation across the buckets, the EBVs are a material fraction of the spread. For example, among the near-term deep in-the-money calls, the average bid-ask spread is $0.72 and the average conditional EBV is $0.35, so the boundary violation represents nearly half of the spread.

A similar pattern occurs in the near-the-money options where the average spread is $0.33 and the conditional EBV is $0.14.

A separate analysis of options that are partitioned into options above $3.00 and below $3.00, and into pilot study options and non-pilot study options is presented in Appendix 1. They reveal significant differences in liquidity across the subgroups, but they generally show the same pattern of EBVs as in Table 1.

Two other considerations about Table 1 should be noted. First, our results are not specific to this time period. In untabulated analyses, we find very similar results in August 2008. Second, the EBVs cannot be easily explained by short sale costs. Asquith, Pathak, and Ritter, (2005), and Kolasinski, Reed, and Ringgenberg (2013) argue that short sale costs are not material for the vast majority of stocks. However, if only a small fraction of stocks have material short sale costs, they cannot explain why almost all near-term, Deep ITM options exhibit EBVs.

### 4.2 Persistence in Exercise Boundary Violations

Boundary violations need not prevent an option holder from selling at a price above intrinsic value as long as there are periods during the day without an EBV. To assess EBV persistence, we select 10:00 a.m. as the initial observation time, because we can be confident that the options market is fully open by that time and all strikes and maturities can be traded freely.[5] If the 10:00 a.m. quote is a boundary violation, we go forward in time and search for the first occurrence of a non-EBV bid, stopping the search process at 4:00 p.m. With this starting time, the maximum duration of a boundary violation is 360 minutes.

[5] Similar results were found when the observations were sampled at 11:00, 12:00, and 2:00.

Table 2 uses the same partitions for option maturities and moneyness as Table 1. The results show a strong persistence in boundary violations. For near term Deep and Mid ITM calls and puts, more than 90% had EBV bids at 10:00 a.m., and these persisted for many hours on average. For example, the average EBV persistence for the near-term Mid ITM contracts is 297 minutes for calls and 296 minutes for puts. A trader who observes an EBV at 10:00 would have to wait, on average, until almost 3:00 pm for the EBV to disappear. A similar pattern is even evident for the long-term options. EBVs are less common in options with more than 4 months to maturity, but when they occur, their magnitude and duration are similar. For example, the violation rate is 59.51% for the Deep ITM calls, but under 1% for the Near ITM contracts. Nevertheless, the persistence of the violations remains high, 266 and 178 minutes, respectively.

We will discuss transactions costs and other impediments that affect option trading in the next section.

## 5. Boundary Violations and Option Trades

### 5.1 Commissions, margins and settlement issues

The optimal exercise strategy for an investor to liquidate an American option position before expiration depends on the actual transactions costs and the other impediments to trading that he faces. Securities firms catering to retail traders charge a wide range of commissions and fees. We discuss how two of the largest handle early exercises.

Interactive Brokers (IB), one of the lowest cost firms, provides full electronic information and execution facilities online. There is no charge for exercising an option, which can be done at any time simply by clicking on a computer screen. When the "exercise" button is clicked, IB immediately places the stock in the investor's account and it can be sold in the market. Within a

few seconds the option holder is able to see the market stock price, exercise his call options, and sell the stock to lock in the current intrinsic value. IB's commission for a stock trade is $0.005 per share ($1 minimum). Thus, an investor can exercise his American calls at any time and receive the intrinsic value at the current stock price, for a total cost of 50 cents per contract (the $1 minimum corresponds to 200 shares, i.e., two option contracts). Exercise of a put that requires first buying the stock to be delivered is similar, and it is even cheaper if the put is protecting stock that is already owned, since no stock trade is needed. Commissions at Charles Schwab, another large low-cost retail broker are somewhat higher, but still low enough that they are easily outweighed by even a very small EBV.

The alternative strategy of holding call options unexercised, but getting rid of their market exposure by delta-hedging with the stock through expiration day is more costly than early exercise. The commission to trade the stock in setting up the hedge will be about the same as to sell out the stock acquired by exercising a call, but depending on the path of stock prices, maintaining delta neutrality can entail actively rebalancing the hedge over time and many more stock trades. In addition, maintaining a short position in the stock requires paying the stock borrow fee and posting collateral, initially equal to 50% of the stock value, in a margin account for the remaining life of the option. This strategy may be appropriate for the holder of a European call who otherwise would need to sell at the EBV bid, but it will normally be dominated by exercise for an American option.[6]

[6] Delta hedging, in theory, allows recovery of an option's full value, not just intrinsic value, which makes it better than exercise in a frictionless world. The intrinsic value is embedded in the difference between the current stock price and the strike, but replication of the optionality value comes from (frictionless) continuous rebalancing of the hedge. Frequent rebalancing in the real world quickly leads to large transactions costs, and it does not fully eliminate risk. In most cases, exercise of an American option will be cheaper, easier, and less risky than attempting to exit the position synthetically by hedging away its market exposure.

### 5.2 Boundary Violations and Option Trades

An investor who sells an option at a price below the exercise boundary is (perhaps unknowingly) leaving money on the table. A natural question to ask is how often does that occur? Although the displayed quotes exhibit sustained boundary violations, investors might either exercise their options instead of selling or trade strategically by picking the right times to avoid, or at least minimize, the boundary violations. To see whether option traders avoid the inferior prices associated with the worst boundary violations, we examine all option trades in our sample that occur above, and below, the boundary violation price, using the bid and ask quotes in effect at the exact time of the trade.[7] For trades below the EBV price, we compute the dollar amount of the deviation from the EBV price.

Table 3 provides summary statistics for option trades in our sample. Option exercises, by definition, are excluded. Note that the relative weighting in this table is by the number of transactions, unlike Tables 1 and 2 in which each option has equal weight. If one option series has 10 trades in a day and the second has only one, the first option will generate 10 transaction observations, while the second option generates just one.

Panel (a) shows EBV trading of calls, and Panel (b) of puts, using the same maturity and moneyness breakdown as before. Panel (c) aggregates across categories to report the total deviation from selling at EBV prices, which totals about $39 million for the month and

[7] It might seem reasonable to measure execution quality by the difference between the trade price and the quote midpoint and then examine whether execution quality improves when the boundary violations are large, because it becomes more possible for investors to trade inside the spread. Unfortunately, this idea does not work because investors trying escape EBVs can be expected to try placing limit orders inside the marketmakers' spread, which alters the recorded best bid price. We argue below that in an EBV it may be more appropriate to treat the option's intrinsic value as the market's best bid.

represents an upper bound on the opportunity costs associated with the failure to exercise. Most of this was due to call option trades, about $37.2 million versus $1.8 million from puts.

In Panel (a) dollar EBV deviations from trading calls occur in each sub-group, and range from three to nine cents per share. The largest dollar apparent losses, however, are concentrated in the near term options, as one expects, because EBVs are larger and more prevalent. Among the near-term, Mid ITM calls, 12.13% of trades and 28.06% of volume occurred below the boundary price. This corresponds to an opportunity loss of $515.25 per trade.

It is important to note that selling at an EBV price can occur from an investor either closing out an existing long position, or from establishing a new short position. Unfortunately, our data cannot distinguish between these two motives. In either case, it is difficult to justify selling at the EBV price because it gives the corresponding dealer the ability to exercise the option for a profit.[8] Consequently, our estimates of the dollar loss from EBV trades reflect a mixture from both motives and should be viewed as an upper bound estimate.

Yet even among options that exhibit EBVs more than 90% of the time, only 12% or so of the trades are done at EBV prices. Interestingly, a higher proportion of trading volume than of trades is done at EBVs, which means those trades are larger than average. A possible reason for this is that call exercise followed by selling the shares in the market depends on adequate liquidity in the underlying stock. The average EBV trade size for near term Mid ITM options is 130 contracts, which corresponds to 13,000 shares, while the average EBV is only $0.04. Depending on the stock, it might well be that dumping 13,000 shares on the market at once could

[8] One possible reason for establishing a short position at an EBV price is to circumvent short sell rules. The SEC is critical of these strategies. See SEC Administrative Proceeding 3-14847.

knock the price down at least 4 cents. The long maturity Mid ITM calls, which averaged nearly 1500 contracts per trade but very few exercises, as we will see in the next section, would certainly be consistent with this explanation. In contrast, put options exhibit a much smaller fraction of EBV trades and volume. Although the boundary violations for several of the put sub-samples are around $0.40 per share, only a very few such trades were done, which makes the overall loss much smaller.

The summary of these results in Panel (c) shows that investors generally avoided selling at sub-optimal prices. Of all in-the-money option trades, only 11.52% of trading volume and 2.01% of trades occurred below the boundary price. On average, the transaction price was about $0.04 below the exercise boundary price, corresponding to a loss of about $521.81 per trade. The evidence suggests that investors adjust their trading behavior to the boundary violations, although it is difficult to prove unambiguously. Comparing the average EBV for trades against the average EBV quotes for the same option categories in Table 1 reveals that investors who did trade managed to avoid the worst of the boundary violations, consistent with getting some price improvement on their trades. Given how pervasive boundary violations were in the quotes, surely more than 2% of trades and 11.5% of volume would be below the boundary price if traders were not paying attention to EBVs. Muravyev and Pearson (2014) suggest an alternative explanation. They show that the underlying stock price can exhibit considerable variation without triggering a change in the option quotes and investors incorporate this information to strategically time their trades to lower their effective trading costs. This is consistent with observing a smaller EBV from actual trades, than from option quotes.

Nevertheless, a one-month loss of $39 million from sub-optimal trades is a material amount and at least some of it could have been saved by exercising the options instead of selling them. By

comparison, the opportunity loss from these trades exceeds the losses documented in Pool, Stoll, and Whaley (2008) from the failure to exercise call options before ex-dividend days (about $4 million per month), and the losses reported in Barraclough and Whaley (2012) from the failure to exercise put options (about $13 million per month).

## 6. Boundary Violations and Option Exercises

Section 4 established that exercise boundary violations are frequent, persistent, and economically significant. In this section we examine how boundary violations affect exercise decisions.

Previous studies reported evidence of early exercise among exchange-traded American-style options but without focusing on EBVs.[9] Given that there appears to be little awareness of the significance of boundary violations in the literature and the established theory emphasizes that early exercise is sub-optimal, it is an open question whether these exercises are mostly mistakes or rational responses to pervasive EBVs.

To examine this question, we analyze the option exercises during our sample period. The available data report only the total number of contracts exercised on a given day, so 20 contracts could be one exercise for all 20, or 20 exercises for 1 contract each. We therefore define the occurrence of one or more exercises for a given contract on a particular day as an Exercise Event.

Customers and market makers have different reasons for holding or exercising in-the-money option positions. A customer with Deep ITM calls most likely bought them for lower prices at

[9] Examples include Diz and Finucane (1993), Overdahl and Martin (1994), Finucane (1997), Engstrom (2002), Pool, Stoll, and Whaley (2008), Hao, Kalay, and Mayhew (2010), Barraclough and Whaley (2012), and Jensen and Pedersen (2015).

some earlier time and may now want to exit the position and take her profit. A market maker rarely has a long term investment motive and will typically acquire an ITM option position by taking the opposite side of a customer's trade. If he buys calls or puts at an EBV price, he may well simply exercise immediately and take the arbitrage profit, rather than holding illiquid high priced options in inventory. Alternatively, he may retain the options if there is a reasonable chance to sell them at his ask price before too long, especially if the delta and other Greek letter exposures are useful in hedging the rest of his trading book. A customer with deep in-the-money puts may well be holding them to protect the value of a long position in the underlying stock. Since liquidating the puts would ruin the hedge, she may be more willing to hold onto them through expiration, even when theory might suggest early exercise would be optimal.

Summary statistics for exercise events are presented in Table 4. None of these exercises should be dividend related, since we exclude all observations from the day before an ex-dividend day. We also exclude the final day before expiration in order to look only at early exercise.

Panel (a) shows that early exercise of calls unconnected to dividend payout is frequent. In March 2010 there were 10,334 exercise events involving 950,683 contracts. As expected, exercise is related to option maturity and moneyness. Near term calls in the Deep and Mid ITM categories experience the most exercises, by both customers and market makers. Customers have more exercise event days but exercise fewer contracts than market makers overall.[10]

Panels (b)-(d) of Table 4 provide the exercise statistics for put options. Unlike calls, if an American put in a frictionless market goes deep enough in-the-money, it becomes rational in

[10] Barraclough and Whaley (2012) also found that market makers exhibit early exercise behavior that runs counter to conventional theory.

theory to exercise it early. For any given put, on every day t there is an early exercise stock price $S_t^*$ such that if the stock price is below $S_t^*$, the put is worth less than its intrinsic value and it should be exercised. For each put observation in our data, we compute the critical stock price for (theoretically) rational early exercise in a Binomial model. The put is then classified as one that should, or should not, be exercised early. Note that exercise when the stock is below $S_t^*$ does not necessarily mean the investor followed the theoretically optimal strategy, which requires exercising the put the first time the boundary is breached. Also, it is important to recognize that there is no direct connection between the critical stock price and an EBV. A put may be in the range where its fair value is above intrinsic value and theoretically it should not be exercised, yet if the best available bid in the market is an EBV, the option holder who must liquidate still does better by exercising the put than by selling it.

Panel (b) shows that there were fewer early exercises of put options than call options, 4324 versus 10,344 exercise events, although they exhibit similar patterns across maturity and moneyness as in Panel (a). One reason for the difference in exercises, as shown in Table 1, is that for the near-term Deep and Mid ITM contracts, there were simply many more quotes for calls than for puts.[11] The ratio of market maker exercises to total exercises is also higher for puts (78.8%) than for calls (58.8%). This would be consistent with the idea that customers may be relatively more reluctant to exercise puts that are hedging a stock position. There is little exercise of either long maturity calls or puts.

[11] Exchanges introduce option strikes around the current underlying stock price, and as the stock price moves, new strikes are introduced while existing options may go deep in or out of the money. A moderate advance in the stock market in the months before March 2010 left many of the previously introduced calls deep in-the-money and the corresponding puts out of-the-money (and out of our sample).

Comparing Panels (c) and (d), put option exercises are divided evenly between those that should have been exercised early in theory and those that should not. Of the 267,604 total put exercises, just 51.7% occurred when the stock price was below the early exercise price barrier. Panel (c) shows that, relative to customers, market makers were more likely to exercise when it was theoretically correct to do so.

### 6.1 Estimating the Likelihood of Early Exercise

The previous section demonstrated that despite being irrational in theory, early exercise is common in the real world. This section investigates early exercise in a more structured framework by fitting logit models for the probability of option exercise. The sample consists of all individual options that have either positive trading volume or a change in open interest on a given day. We exclude any options with a zero bid price and require each option series to have at least 385 valid quote minutes per day.

In our logit specifications, the likelihood of early exercise is related to the moneyness and maturity. Our operational measure of moneyness is related to the probability that the option will still be in-the-money at maturity and this is proportional to $\sqrt{T}$. We measure this by $(X/S)/\sqrt{T}$ for call options and by $(S/X)/\sqrt{T}$ for put options. Scaling the moneyness by $\sqrt{T}$ makes the measure more comparable across maturities. Since there is a wide variation in the daily minimum EBV, we use an indicator variable that is one if the daily minimum is at least five cents, and zero otherwise. For the persistence of boundary violations, we compute the number of minutes during the trading day that an option exhibited a boundary violation. We also include the option's volume, open interest, and implied volatility as control variables. For the put models, we include

an additional indicator variable that is one if the stock price is below the critical level where the put should be exercised early, and zero otherwise.

The results are presented in Table 5. The first three columns contain the estimates for call options, while the last three columns provide the put option estimates. As expected, the results in column 1 show that the likelihood of early exercise for call options is strongly related to our moneyness measure. The interesting result, however, is that both EBV variables are highly significant. All else constant, the existence of EBVs and their duration, strongly predict early exercise. Of the control variables, open interest and daily trading volume are positively related to the exercise likelihood, while the implied volatility is insignificant.

The results are similar for the put options, as shown in column 4. Both EBV variables are again highly significant as is the early exercise price boundary variable, suggesting that exercise is more likely when the price is below the early exercise price barrier. Open interest is positively related to the exercise likelihood while the implied volatility and trading volume are insignificant.

One notable difference between the call and put results is that the moneyness coefficient for the puts is insignificant. One possible reason for this is that these puts have two desirable hedging properties: they have negative exposure to the underlying stock and virtually no gamma. If the puts are being used to hedge a long position in the underlying stock, then their exercise would require delivering the stock and liquidating the hedge, which may well not fit with the investor's long-term investment plan.

Table 5 shows some interesting differences between customer and market maker exercises. Comparing columns 2 and 3, the coefficients for the EBV variables are significantly larger for

the market makers, suggesting that they respond more aggressively to EBVs than do the customers. Similarly, the coefficients in columns 5 and 6 show that market makers are more responsive than customers to both the EBVs and the early put exercise price barrier.

A potential concern is that our results may influenced by liquidity differences related to price levels. As discussed earlier, the minimum bid-ask spread is $0.10 for option prices above $3.00 and $0.05 for prices below $3.00. As a robustness check, we tried adding two indicator variables to the logit model from Table 5. The first variable was one if the option price was between $2.50 and $3.00, and zero otherwise, while the second was one if the price was between $3.00 and $3.50 and zero otherwise. Five of the six sub-samples failed to reject the hypothesis of equality. We also reestimated the Table 5 specifications for the pilot and non-pilot samples separately. The relation between EBVs and likelihood of early exercise for both sub-samples was similar to the results from Table 5.

Overall, our results suggest that early exercise is a rational response to EBVs and we need to revisit the claim that American-style options on non-dividend paying stocks should not be exercised before maturity. Further, our findings illustrate the importance of incorporating EVBs into option pricing models. If option investors are aware of EBVs and try to avoid or minimize them, this will be reflected in option liquidity and the early exercise premium. We discuss these issues in the next section.

## 7. EBVs and the Measurement of the Early Exercise Premium, Implied Volatilities, and Market Liquidity

### 7.1 Boundary Violations and the Early Exercise Premium

Boundary violations have a material impact on the early exercise decision in real world markets. They make exercise the rational way to unwind an in-the-money American option position for a much broader range of maturity and moneyness conditions than would be expected in a frictionless market. The economically meaningful increase in liquidity that American exercise provides relative to a comparable European option should be reflected in an early exercise premium that is on top of any premium related to future dividend payout, and empirical research supports this view.[12]

Developing an alternative American option pricing model to take EBVs fully into account is beyond the scope of the current paper. However, it is not hard to generate premia for American exercise within a simulation framework such as in Figlewski (1989). We have done so in a "back-of-the-envelope'" exercise, beginning with a standard diffusion process for the stock price and computing the resulting Black-Scholes prices but adding a constant bid-ask spread centered on the Black-Scholes value to any option trade, which induces EBVs for Deep ITM calls. To create a need to liquidate the option position before maturity, we add a survival function in the form of a simple exponential decay, which specifies, for each future date, the probability the position must be unwound--either sold at the market bid price or exercised. For each liquidation date, we compute the incremental value gained from the ability to exercise versus having to sell at the market bid. The discounted expected value of this gain represents the value of American exercise.

[12] Valkanov, Yadav, and Zhang (2011) and Dueker and Miller (2003) found that theoretical premia for dividend-related early exercise are too small to explain the actual premiums observed in the market.

As an illustration, we considered the additional value of American exercise using the following parameter values, that are roughly comparable to those found in our data sample: Stock price = 100; Exercise price = 90; Maturity = 20 trading days; Volatility = 35%; Bid-Ask spread = $1.50 (13.6% of initial call price); Interest: 5%; Probability of liquidation before expiration: 75%, with constant intensity per day; no dividends. Under these assumptions, the liquidity value of American exercise is about $0.19.

For comparison, a 20-day American call on a stock that will pay a quarterly of $0.50 during the same period (2% annual yield), would be worth about $0.44 more than the equivalent European call. In this case, an American call on a non-dividend paying stock that can avoid a future EBV by early exercise has a liquidity-related premium over the European call about half as large as the dividend-related premium for the first call.

### 7.2 Boundary Violations and Mark-to-Market Option Valuation

It is customary to consider an option's market price to be the midpoint between the current best bid and ask prices. But one might argue that when the market bid price for an American option is an EBV, the lowest portion of the bid-ask spread becomes somewhat irrelevant to the value of the option, since the investor can realize a larger value by exercise. The most obvious case is when the bid is so low that the midpoint is also below intrinsic value. For the purpose of setting the required margin on an option position, it is questionable whether it is appropriate to value the option at the bid-ask midpoint when a higher value could be obtained by immediate exercise. In such a case, it could make sense to replace the EBV bid quote with the option's intrinsic value.

### 7.3 Boundary Violations and Implied Volatilities and Other Implicit Parameters

Boundary violations have implications for the calculation of implied volatilities and option hedge parameters, as well as for the implicit parameters which arise in more elaborate models that extend Black-Scholes.

An option's implied volatility is generally computed from the bid-ask midpoint, which assumes the midpoint is an unbiased estimate of the true option price. But since a call must be priced above intrinsic value in a theoretical model, implied volatility is undefined in the Black-Scholes model at an EBV price, as are the implied parameters in more sophisticated pricing models. When the quote midpoint is also below intrinsic value, IV and other implied parameters are unavailable. In this case, also, it might be better to replace the EBV bid price with the option's intrinsic value.

**7.4 Boundary violations and liquidity measures.**

Boundary violations have important implications for measures of transaction costs. One interpretation of a boundary violation is that the traditional bid-ask spread is too large. A more realistic 'adjusted' estimate of the bid-ask spread facing investors would be the difference between the ask price and the maximum of either the bid price, or the intrinsic value.

The difference between the traditional and boundary adjusted spreads can be substantial. For the deep in-the-money near-term call options in Table 1, the average traditional spread is about $0.72 while the boundary adjusted spread is only $0.38. The difference is smaller, but still substantial, for near-term Near ATM calls: $0.33 for the traditional spread and $0.26 for the boundary adjusted spread.

Many studies have used option bid and ask prices to make inferences about regulatory issues, market making costs, and option price dynamics. While it is uncertain how much these analyses are affected by boundary violations, given their prevalence, the violations are surely a material component of the data. An important implication is that it would be comparatively easy for a retail broker to offer option investors significant price improvement simply by providing a real time comparison of the payoffs to exercising vs. selling their options. Alternatively, it should be straightforward for an algorithm to automatically route the customer's order to the best alternative, which would include early exercise. In our conversations with option broker-dealers some confirmed that they inform their customers when their trades would be sub-optimal.

## 8. Conclusion

Exercise boundary violations (EBVs) occur when an option bid price is low enough that an investor would be better off exercising his option instead of selling it. This paper is the first systematic analysis of such boundary violations. We first established that boundary violations are common, persistent, and economically significant. Previous research documented the existence of boundary violations; our contribution is to show how important they are in the data.

Second, boundary violations and the resulting option trade prices are inconsistent with the assumptions made in traditional option pricing models, in which option quotes and trades should never be below intrinsic value. Yet, these events occur frequently and systematically. As is being increasingly found in other areas of finance, liquidity issues for options in the real world need not just produce "noise" around theoretical relationships, which may be ignored in empirical work. Liquidity problems can lead to quite different behavior in practice than what

may be optimal in theory. New models that can predict market behavior consistent with the data are needed.

Third, the analysis of exercises versus trades at EBV prices revealed that investors do appear to respond at least somewhat rationally to the size and prevalence of EBV quotes. Numerous previous studies have noted that options are often exercised early when conventional theory says early exercise is sub-optimal, and have concluded that investors do not behave rationally. By contrast, in the presence of boundary violations exercise is typically the rational choice, and we find that many investors make this choice: the likelihood of early exercise is significantly related to the existence and duration of exercise boundary violations.

Extensive exercise boundary violations are evidence of the important and largely unexamined role of liquidity in options markets, with numerous implications for research in options microstructure and pricing.

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**Table 1**
**Summary Statistics for Option Quotes**

This table contains summary statistics for all option quotes during March 2010. Quotes from options with less than 180 observations per day or quotes with a zero bid price are excluded. Total EBV quotes is the number of quotes that exhibit exercise boundary violations. % Quotes with EBV is the percentage of quotes that exhibit boundary violations. % Options with EBV is the fraction of options that have one or more boundary violations during a trading day. Unconditional EBV is the mean boundary violation, averaged across all quotes. Conditional EBV is the mean boundary violation averaged across all quotes with boundary violations. EBV Adjusted Spread is the bid-ask spread computed from the ask price and the maximum of either the bid price or exercise boundary price. In the column headings for calls, Deep ITM corresponds to call options with a strike price/stock price ratio of between .5 and .7. Mid refers to a stock price price/strike price ratio between .7 and .9, and Near refers to a ratio between .9 and 1.0. In the column headings for puts Deep ITM corresponds to put options with a strike price/stock price ratio of between 1.3 and 1.5. Mid refers to a stock price price/strike price ratio between 1.1 and 1.3, and Near refers to a ratio between 1.0 and 1.1.

**Panel (a): Call Options**

| **Maturity** | **< 1 Month** | **< 1 Month** | **< 1 Month** | **2-4 Months** | **2-4 Months** | **2-4 Months** | **> 4 Months** | **> 4 Months** | **> 4 Months** |
|---|---|---|---|---|---|---|---|---|---|
| **Moneyness** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** |
| Total Quotes | 24,233,775 | 39,210,622 | 20,603,553 | 41,276,612 | 62,774,801 | 34,708,238 | 56,578,410 | 69,881,531 | 36,786,243 |
| Total EBV Quotes | 23,925,783 | 36,374,930 | 7,538,395 | 38,676,453 | 34,011,298 | 1,804,803 | 33,821,214 | 11,251,916 | 332,710 |
| % Quotes with EBV | 98.73% | 92.77% | 36.59% | 93.70% | 54.18% | 5.20% | 59.78% | 16.10% | 0.90% |
| % Options with EBV | 99.00% | 98.21% | 57.40% | 98.13% | 73.15% | 12.67% | 74.98% | 26.80% | 2.28% |
| Option Bid Price | $15.90 | $7.67 | $2.27 | $16.20 | $7.85 | $3.14 | $19.71 | $10.53 | $5.77 |
| Option $ Spread | $0.72 | $0.58 | $0.33 | $0.85 | $0.61 | $0.34 | $1.02 | $0.73 | $0.46 |
| Option % Spread | 9.26% | 15.05% | 31.71% | 10.21% | 13.90% | 20.23% | 9.29% | 12.82% | 15.57% |
| Unconditional EBV | $0.34 | $0.25 | $0.07 | $0.37 | $0.16 | $0.01 | $0.27 | $0.06 | $0.00 |
| Conditional EBV | $0.35 | $0.26 | $0.14 | $0.38 | $0.24 | $0.18 | $0.39 | $0.28 | $0.35 |
| EBV Adjusted $ Spread | $0.38 | $0.33 | $0.26 | $0.48 | $0.45 | $0.33 | $0.75 | $0.67 | $0.46 |
| EBV Adjusted % Spread | 4.85% | 8.60% | 26.11% | 5.75% | 10.26% | 18.90% | 6.61% | 11.20% | 15.02% |

## Panel (b): Put Options

| **Maturity**<br>**Moneyness** | **< 1 Month**<br>**Deep** | **< 1 Month**<br>**Mid** | **< 1 Month**<br>**Near** | **2-4 Months**<br>**Deep** | **2-4 Months**<br>**Mid** | **2- 4 Months**<br>**Near** | **> 4 Months**<br>**Deep** | **> 4 Months**<br>**Mid** | **> 4 Months**<br>**Near** |
|---|---|---|---|---|---|---|---|---|---|
| Total Quotes | 12,146,014 | 27,155,723 | 19,520,667 | 22,905,184 | 46,400,832 | 33,022,460 | 33,962,640 | 54,848,071 | 35,199,807 |
| Total EBV Quotes | 11,788,040 | 24,348,361 | 7,062,711 | 17,565,888 | 19,510,454 | 1,386,092 | 8,931,651 | 3,301,589 | 208,339 |
| % Quotes with EBV | 97.05% | 89.66% | 36.18% | 76.69% | 42.05% | 4.20% | 26.30% | 6.02% | 0.59% |
| % Options with EBV | 99.27% | 97.14% | 55.96% | 90.09% | 62.73% | 10.35% | 43.51% | 13.00% | 1.40% |
| Option Bid Price | $12.81 | $6.90 | $2.10 | $14.15 | $7.45 | $3.05 | $19.51 | $10.75 | $5.89 |
| Option $ Spread | $0.65 | $0.58 | $0.34 | $0.76 | $0.59 | $0.34 | $0.84 | $0.63 | $0.43 |
| Option % Spread | 12.23% | 17.48% | 35.09% | 11.48% | 14.89% | 20.27% | 8.94% | 11.37% | 14.04% |
| Unconditional EBV | $0.30 | $0.24 | $0.07 | $0.26 | $0.12 | $0.01 | $0.11 | $0.03 | $0.00 |
| Conditional EBV | $0.31 | $0.25 | $0.14 | $0.30 | $0.22 | $0.20 | $0.30 | $0.33 | $0.39 |
| EBV Adjusted $ Spread | $0.35 | $0.34 | $0.27 | $0.50 | $0.47 | $0.33 | $0.73 | $0.60 | $0.42 |
| EBV Adjusted % Spread | 6.52% | 10.46% | 29.20% | 7.35% | 11.67% | 19.09% | 7.18% | 10.27% | 13.56% |

## Table 2

## Duration of Boundary Violations

This table examines the persistence of exercise boundary violations for all options. Option Days represents the number of individual options with positive option trading volume on day t and complete intraday quote data for day t. 10:00 a.m. EBV Frequency is the fraction of quotes that show exercise boundary violations when measured at 10:00 a.m. Avg Size of 10:00 a.m. EBV is the mean exercise boundary violation at 10:00 a.m., conditional on observing a boundary violation at 10:00 a.m. Avg Duration of EBV is the average number of minutes after 10:00 a.m. the boundary violation persists. In the column headings for calls, Deep ITM corresponds to call options with a strike price/stock price ratio of between .5 and .7. Mid refers to a stock price price/strike price ratio between .7 and .9, and Near refers to a ratio between .9 and 1.0. In the column headings for puts Deep ITM corresponds to put options with a strike price/stock price ratio of between 1.3 and 1.5. Mid refers to a stock price price/strike price ratio between 1.1 and 1.3, and Near refers to a ratio between 1.0 and 1.1.

### Panel (a): Calls

| **Maturity** | **< 1 Month** | **< 1 Month** | **< 1 Month** | **2-4 Months** | **2-4 Months** | **2-4 Months** | **> 4 Months** | **> 4 Months** | **> 4 Months** |
|---|---|---|---|---|---|---|---|---|---|
| **Moneyness** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** |
| Option Days | 64,497 | 104,737 | 55,087 | 110,416 | 167,730 | 92,759 | 151,442 | 186,581 | 98,357 |
| 10:00 a.m. EBV Frequency | 99.56% | 93.43% | 35.77% | 93.96% | 53.51% | 4.92% | 59.51% | 15.84% | 0.93% |
| Avg Size of 10:00 a.m. EBV | $0.38 | $0.29 | $0.19 | $0.40 | $0.30 | $0.28 | $0.46 | $0.39 | $0.41 |
| Avg Duration of EBV | 348 | 297 | 189 | 313 | 235 | 157 | 266 | 228 | 178 |

### Panel (b): Puts

| **Maturity** | **< 1 Month** | **< 1 Month** | **< 1 Month** | **2-4 Months** | **2-4 Months** | **2-4 Months** | **> 4 Months** | **> 4 Months** | **> 4 Months** |
|---|---|---|---|---|---|---|---|---|---|
| **Moneyness** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** |
| Option Days | 32,804 | 72,812 | 52,309 | 61,878 | 124,258 | 88,316 | 91,364 | 146,771 | 94,144 |
| 10:00 a.m. EBV Frequency | 97.31% | 90.46% | 36.46% | 77.41% | 42.65% | 4.35% | 26.64% | 6.27% | 0.55% |
| Avg Size of 10:00 a.m. EBV | $0.33 | $0.29 | $0.20 | $0.34 | $0.29 | $0.30 | $0.41 | $0.45 | $0.46 |
| Avg Duration of EBV | 335 | 296 | 196 | 274 | 216 | 154 | 207 | 188 | 219 |

## Table 3
## Summary Statistics for Option Trades

This table provides summary statistics for all in-the-money option trades during March 2010. Total Trades is the aggregate number of trades that occurred for each sub-group. % Trades<EBV is the percentage of trades that occur at a price below the exercise boundary price. % Volume<EBV is the percentage of volume that occurs at a price below the exercise boundary price. Trade Size is the average number of contracts for trades that occur at a price below the exercise boundary price. $ Deviation per share is the average dollar amount per share from selling at a price below the exercise boundary price. $ Deviation per trade is the average dollar amount per trade from selling at a price below the exercise boundary price. $ Deviation Total is the cumulative amount for each sub group from selling at a price below the exercise boundary price. In the column headings for calls, Deep ITM corresponds to call options with a strike price/stock price ratio of between .5 and .7. Mid refers to a stock price price/strike price ratio between .7 and .9, and Near refers to a ratio between .9 and 1.0. In the column headings for puts, Deep ITM corresponds to put options with a strike price/stock price ratio of between 1.3 and 1.5. Mid refers to a stock price price/strike price ratio between 1.1 and 1.3, and Near refers to a ratio between 1.0 and 1.1.

**Panel (a): Call Options**

| **Maturity** / **Moneyness** | **< 1 Month** **Deep** | **< 1 Month** **Mid** | **< 1 Month** **Near** | **2-4 Months** **Deep** | **2-4 Months** **Mid** | **2-4 Months** **Near** | **> 4 Months** **Deep** | **> 4 Months** **Mid** | **>4 Months** **Near** |
|---|---|---|---|---|---|---|---|---|---|
| Total trades | 19,030 | 179,246 | 1,148,077 | 17,433 | 161,612 | 702,221 | 38,459 | 119,368 | 212,439 |
| % Trades<EBV | 12.10% | 12.13% | 2.21% | 12.40% | 3.79% | 0.14% | 5.37% | 0.82% | 0.00% |
| % Volume<EBV | 22.26% | 28.06% | 14.27% | 40.73% | 17.95% | 0.57% | 18.24% | 37.30% | 0.16% |
| Trade size | 31.21 | 130.29 | 194.70 | 122.51 | 138.20 | 72.58 | 206.87 | 1457.57 | 108.78 |
| $ Deviation per share | $0.09 | $0.04 | $0.03 | $0.05 | $0.04 | $0.06 | $0.04 | $0.03 | $0.04 |
| $ Deviation per trade | $282.31 | $515.25 | $560.70 | $638.69 | $536.64 | $454.09 | $835.67 | $4372.59 | $482.89 |
| $ Deviation total | $649,884 | $11,204,146 | $14,211,482 | $1,380,211 | $3,284,747 | $446,829 | $1,725,656 | $4,258,904 | $30,422 |

**Panel (b):**
**Put Options**

| **Maturity**<br>**Moneyness** | **< 1 Month**<br>**Deep** | **< 1 Month**<br>**Mid** | **< 1 Month**<br>**Near** | **2-4 Months**<br>**Deep** | **2-4 Months**<br>**Mid** | **2-4 Months**<br>**Near** | **> 4 Months**<br>**Deep** | **> 4 Months**<br>**Mid** | **>4 Months**<br>**Near** |
|---|---|---|---|---|---|---|---|---|---|
| Total trades | 5,732 | 46,101 | 542,766 | 5,770 | 48,804 | 307,684 | 12,297 | 47,104 | 108,016 |
| % EBV Trades | 15.49% | 8.69% | 1.18% | 6.41% | 2.11% | 0.05% | 0.02% | 0.11% | 0.01% |
| % EBV Volume | 21.87% | 6.52% | 1.05% | 3.03% | 2.63% | 0.08% | 0.24% | 0.12% | 0.00% |
| Trade size | 26.14 | 13.51 | 16.24 | 11.90 | 30.38 | 33.55 | 11.58 | 25.40 | 6.78 |
| $ Deviation per share | $0.05 | $0.07 | $0.06 | $0.13 | $0.11 | $0.44 | $0.38 | $0.40 | $1.01 |
| $ Deviation per trade | $126.03 | $89.09 | $96.19 | $156.69 | $334.63 | $1467.70 | $444.54 | $1027.19 | $683.56 |
| $ Deviation total | $111,917 | $356,733 | $617,069 | $57,976 | $344,330 | $239,235 | $25,339 | $53,414 | $6,152 |

**Panel (c): All Options**

| | **Calls & Puts** | **Calls** | **Puts** |
|---|---|---|---|
| Total trades | 3,722,159 | 2,597,885 | 1,124,274 |
| % Trades<EBV | 2.01% | 2.38% | 1.16% |
| % Volume<EBV | 11.52% | 14.81% | 0.98% |
| Trade size | 148.50 | 176.09 | 17.30 |
| $ Deviation per share | $0.04 | $0.03 | $0.08 |
| $ Deviation per trade | $521.81 | $602.20 | $139.54 |
| $ Deviation total | $39,004,446 | $37,192,281 | $1,812,165 |

## Table 4

### Summary Statistics for Option Exercises

This table provides summary statistics for option contracts exercised before maturity during March 2010. An exercise event is when one or more contracts on an individual option are exercised on day t. Total Contracts Traded is the total trading volume on days when exercise events occurred. In Panel (c), Total contracts/total volume for puts that should be exercised is the ratio of exercise volume to trading volume on exercise event days. Near Term options have one month or less to maturity, Medium Term have two to four months, and Long term have at least five months to expiration. The Deep in-the-money calls have a strike price/stock price between .5 and .7, Mid options have a ratio between .7 and .9, and Near has a ratio between .9 and 1.0. For put options, Deep corresponds to ratio of between 1.3 and 1.5, Mid has a ratio between 1.1 and 1.3, and Near refers to a ratio between 1.0 and 1.1.

**Panel (a): Call Options**

| **Maturity Category** | **<1 Month** | **< 1 Month** | **<1 Month** | **2-4 Months** | **2-4 Months** | **2-4 Months** | **>4 Months** | **>4 Months** | **>4 Months** | **Total** |
|---|---|---|---|---|---|---|---|---|---|---|
| **Strike Price Category** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | |
| Exercise Events Total | 2,289 | 4,893 | 1,313 | 833 | 581 | 62 | 258 | 104 | 11 | 10,344 |
| Exercise Events Customer | 1,108 | 3,402 | 1,106 | 479 | 428 | 47 | 154 | 84 | 8 | 6,816 |
| Exercise Events Market Maker | 1,384 | 1,912 | 285 | 401 | 175 | 17 | 110 | 22 | 3 | 4,309 |
| Contracts Exercised Total | 155,361 | 524,900 | 159,001 | 45,004 | 35,476 | 23,856 | 4,449 | 2,589 | 47 | 950,683 |
| Contracts Exercised Customer | 42,495 | 193,552 | 93,226 | 16,035 | 21,325 | 21,683 | 1,948 | 1,140 | 12 | 391,416 |
| Contracts Exercised Market Maker | 112,866 | 331,348 | 65,775 | 28,969 | 14,151 | 2,173 | 2,501 | 1,449 | 35 | 559,267 |

**Panel (b): All Put Options**

| **Maturity Category** | **<1 Month** | **< 1 Month** | **<1 Month** | **2-4 Months** | **2-4 Months** | **2-4 Months** | **>4 Months** | **>4 Months** | **>4 Months** | **Total** |
|---|---|---|---|---|---|---|---|---|---|---|
| **Strike Price Category** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | |
| Exercise Events Total | 619 | 2,291 | 861 | 257 | 262 | 17 | 9 | 6 | 2 | 4,324 |
| Exercise Events Customer | 170 | 720 | 378 | 48 | 59 | 11 | 4 | 6 | 0 | 1,396 |
| Exercise Events Market Maker | 467 | 1,693 | 566 | 212 | 207 | 7 | 5 | 0 | 2 | 3,159 |
| Contract Exercised Total | 49,079 | 128,659 | 64,186 | 11,590 | 13,178 | 649 | 167 | 75 | 21 | 267,604 |
| Contracts Exercised Customer | 10,733 | 35,829 | 16,607 | 2,047 | 1,927 | 76 | 15 | 75 | 0 | 67,309 |
| Contracts Exercised Market Maker | 38,346 | 92,830 | 47,579 | 9,543 | 11,251 | 573 | 152 | 0 | 21 | 200,295 |

## Panel (c): Put Options that should be exercised early

| Maturity Category | <1 Month | < 1 Month | <1 Month | 2-4 Months | 2-4 Months | 2-4 Months | >4 Months | >4 Months | >4 Months | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| **Strike Price Category** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | |
| Exercise Events Total | 405 | 1,217 | 351 | 82 | 49 | 0 | 1 | 0 | 0 | 2,105 |
| Exercise Events Customer | 100 | 319 | 118 | 11 | 5 | 0 | 0 | 0 | 0 | 553 |
| Exercise Events Market Maker | 317 | 974 | 279 | 72 | 45 | 0 | 1 | 0 | 0 | 1,688 |
| Contract Exercised Total | 39,267 | 64,956 | 30,374 | 2,075 | 1,593 | 0 | 1 | 0 | 0 | 138,266 |
| Contracts Exercised Customer | 6,670 | 15,012 | 6,116 | 102 | 243 | 0 | 0 | 0 | 0 | 28,143 |
| Total Contracts Traded | 26,025 | 105,271 | 69,030 | 2,387 | 2,578 | 935 | 14 | 51 | 0 | 206,291 |
| Total Contracts/Total Volume | 150.88% | 61.70% | 44.00% | 86.93% | 61.79% | 0.00% | 7.14% | 0.00% | | 67.02% |

## Panel (d): Put Options that should not be exercised early

| Maturity Category | <1 Month | < 1 Month | <1 Month | 2-4 Months | 2-4 Months | 2-4 Months | >4 Months | >4 Months | >4 Months | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| **Strike Price Category** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | **Deep** | **Mid** | **Near** | |
| Exercise Events Total | 214 | 1,074 | 510 | 175 | 213 | 17 | 8 | 6 | 2 | 2,219 |
| Exercise Events Customer | 70 | 401 | 260 | 37 | 54 | 11 | 4 | 6 | 0 | 843 |
| Exercise Events Market Maker | 150 | 719 | 287 | 140 | 162 | 7 | 4 | 0 | 2 | 1,471 |
| Contract Exercised Total | 9,812 | 63,703 | 33,812 | 9,515 | 11,585 | 649 | 166 | 75 | 21 | 129,338 |
| Contracts Exercised Customer | 4,063 | 20,817 | 10,491 | 1,945 | 1,684 | 76 | 15 | 75 | 0 | 39,166 |
| Contracts Exercised Market Maker | 5,749 | 42,886 | 23,321 | 7,570 | 9,901 | 573 | 151 | 0 | 21 | 90,172 |

## Table 5
## The Likelihood of Early Option Exercise

This table provides estimates of logit models for the probability that an option is exercised before maturity. The dependent variable is one if an option is exercised on day t and zero otherwise. Moneyness for call options is $(X/S)/\sqrt{T}$. EBV>0_minutes is the number of minutes during the trading day that an individual option exhibits a boundary violation. EBV>.05 is an indicator variable if the minimum boundary violation for an individual option during day t is at least .05, and zero otherwise. Option_volume is the number of contracts on an individual option that are traded on day t. Option_openinterest is the open interest for an individual option at the close of day t. Option_IV is the implied volatility of the option at the close of day t. The moneyness variable for put options is defined as $(S/X)/\sqrt{T}$. Early exercise is an indicator variable that is one if the price of the stock is sufficiently low that early exercise is optimal. The t-statistics for all models are adjusted for conditional heteroskedasticity and reported in parentheses.

| | Total Call Exercises | Customer Call Exercises | Market Maker Call Exercises | Total Put Exercises | Customer Put Exercises | Market Maker Put Exercises |
|---|---|---|---|---|---|---|
| **Constant** | -6.617 | -6.662 | -8.794 | -7.667 | -7.919 | -8.843 |
| | (-111.94) | (-113.16) | (-44.25) | (-48.395) | (-55.94) | (-34.08) |
| **Moneyness** | 0.127 | 0.129 | 0.067 | -0.018 | 0.099 | -0.036 |
| | (34.03) | (32.84) | (14.25) | (-0.313) | (12.07) | (-5.40) |
| **Minutes EBV > 0** | 0.012 | 0.011 | 0.016 | 0.015 | 0.012 | 0.018 |
| | (71.92) | (65.21) | (29.43) | (34.535) | (30.37) | (25.34) |
| **EBV >= .05** | 1.066 | 0.681 | 0.928 | 0.883 | 0.333 | 0.672 |
| | (41.11) | (23.47) | (27.51) | (18.130) | (5.13) | (13.53) |
| **Daily trading volume x 1000** | 0.059 | 0.058 | 0.024 | -0.007 | 0.019 | -0.076 |
| | (2.12) | (2.15) | (2.48) | (-0.466) | (1.31) | (-0.47) |
| **Open interest x 1000** | 0.006 | 0.006 | 0.002 | 0.012 | 0.007 | 0.013 |
| | (3.04) | (3.01) | (0.91) | (5.800) | (2.42) | (4.72) |
| **Implied Volatility** | 0.000 | -0.000 | -0.003 | -0.035 | 0.004 | -0.096 |
| | (-0.06) | (-0.06) | (-0.65) | (-1.362) | (0.11) | (-3.21) |
| **Early exercise boundary** | | | | 1.195 | 0.072 | 1.196 |
| | | | | (25.810) | (0.98) | (22.17) |
| **Pseudo $R^2$** | 0.224 | 0.139 | 0.095 | 0.247 | 0.066 | 0.197 |
| **Observations** | 197,610 | 197,610 | 197,610 | 104,329 | 104,321 | 104,321 |